Example 7b: Electromagnetic RUC Analysis

This example problem involves the use of the new electromagnetic micromechanics capabilities within MAC/GMC 4.0. The basic difference between standard and electromagnetic micromechanics involves the form of the material constitutive equation employed. In the standard micromechanics constitutive equations, stresses and strains are related by the stiffness matrix. In electromagnetic micromechanics, the constitutive equations are expanded,

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \\ D_1 \\ D_2 \\ D_3 \\ B_1 \\ B_2 \\ B_3 \end{bmatrix}^{(\alpha\beta\gamma)} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} & e_{11} & e_{21} & e_{31} & q_{11} & q_{21} & q_{31} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} & e_{12} & e_{22} & e_{32} & q_{12} & q_{22} & q_{32} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} & e_{13} & e_{23} & e_{33} & q_{13} & q_{23} & q_{33} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} & e_{14} & e_{24} & e_{34} & q_{14} & q_{24} & q_{34} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} & e_{15} & e_{25} & e_{35} & q_{15} & q_{25} & q_{35} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} & e_{16} & e_{26} & e_{36} & q_{16} & q_{26} & q_{36} \\ e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} & -\kappa_{11} & -\kappa_{12} & -\kappa_{13} & -a_{11} & -a_{12} & -a_{13} \\ e_{21} & e_{22} & e_{23} & e_{24} & e_{25} & e_{26} & -\kappa_{12} & -\kappa_{22} & -\kappa_{23} & -a_{21} & -a_{22} & -a_{23} \\ e_{31} & e_{32} & e_{33} & e_{34} & e_{35} & e_{36} & -\kappa_{13} & -\kappa_{23} & -\kappa_{33} & -a_{31} & -a_{32} & -a_{33} \\ q_{11} & q_{12} & q_{13} & q_{14} & q_{15} & q_{16} & -a_{11} & -a_{21} & -a_{31} & -\mu_{11} & -\mu_{12} & -\mu_{13} \\ q_{21} & q_{22} & q_{23} & q_{24} & q_{25} & q_{26} & -a_{12} & -a_{22} & -a_{32} & -\mu_{12} & -\mu_{22} & -\mu_{23} \\ q_{31} & q_{32} & q_{33} & q_{34} & q_{35} & q_{36} & -a_{13} & -a_{23} & -a_{33} & -\mu_{13} & -\mu_{23} & -\mu_{33} \end{bmatrix}^{(\alpha\beta\gamma)} \begin{bmatrix} \varepsilon_{11} - \varepsilon_{11}^{I} - \varepsilon_{11}^{T} \\ \varepsilon_{22} - \varepsilon_{22}^{I} - \varepsilon_{22}^{T} \\ \varepsilon_{33} - \varepsilon_{33}^{I} - \varepsilon_{33}^{T} \\ 2\varepsilon_{23} - 2\varepsilon_{23}^{I} - 2\varepsilon_{23}^{T} \\ 2\varepsilon_{13} - 2\varepsilon_{13}^{I} - 2\varepsilon_{13}^{T} \\ 2\varepsilon_{12} - 2\varepsilon_{12}^{I} - 2\varepsilon_{12}^{T} \\ -E_1 - E_1^T \\ -E_2 - E_2^T \\ -E_3 - E_3^T \\ -H_1 - H_1^T \\ -H_2 - H_2^T \\ -H_3 - H_3^T \end{bmatrix}^{(\alpha\beta\gamma)}$$

where $\sigma_{ij}^{(\alpha\beta\gamma)}$ are the stress components, $D_k^{(\alpha\beta\gamma)}$ are the electric displacement components, $B_k^{(\alpha\beta\gamma)}$ are the magnetic flux density components, $\varepsilon_{ij}^{(\alpha\beta\gamma)}$ are the total strain components, $\varepsilon_{ij}^{I(\alpha\beta\gamma)}$ are the inelastic strain components, $\varepsilon_{ij}^{T(\alpha\beta\gamma)}$ are the thermal strain components, $E_k^{(\alpha\beta\gamma)}$ are the electric field components, $E_k^{T(\alpha\beta\gamma)}$ are the thermo-electric field components, $H_k^{(\alpha\beta\gamma)}$ are the magnetic field components, $H_k^{T(\alpha\beta\gamma)}$ are the thermo-magnetic field components, $C_{ij}^{(\alpha\beta\gamma)}$ are the material stiffness components, $e_{kj}^{(\alpha\beta\gamma)}$ are the material piezoelectric components, $q_{kj}^{(\alpha\beta\gamma)}$ are the material piezomagnetic components, $\kappa_{ij}^{(\alpha\beta\gamma)}$ are the material dielectric components, $a_{ij}^{(\alpha\beta\gamma)}$ are the material magnetoelectric components, and $\mu_{ij}^{(\alpha\beta\gamma)}$ are the material magnetic permeability components of a given subcell (denoted by the indices $\alpha\beta\gamma$). The thermal strain and thermal field components are related to a change in temperature from a given reference temperature (i.e., ΔT) by,

$$\begin{bmatrix} \varepsilon_{11}^T & \varepsilon_{22}^T & \varepsilon_{33}^T & 2\varepsilon_{23}^T & 2\varepsilon_{13}^T & 2\varepsilon_{12}^T & E_1^T & E_2^T & E_3^T & H_1^T & H_2^T & H_3^T \end{bmatrix}^{(\alpha\beta\gamma)} = \\ \begin{bmatrix} \alpha_{11} & \alpha_{22} & \alpha_{33} & \alpha_{23} & \alpha_{13} & \alpha_{12} & \zeta_1 & \zeta_2 & \zeta_3 & \psi_1 & \psi_2 & \psi_3 \end{bmatrix}^{(\alpha\beta\gamma)} \Delta T$$

where $\alpha_{ij}^{(\alpha\beta\gamma)}$ are the subcell material coefficients of thermal expansion (CTEs), $\zeta_k^{(\alpha\beta\gamma)}$ are the subcell material pyroelectric constants, and $\psi_k^{(\alpha\beta\gamma)}$ are the subcell material pyromagnetic constants.

For the truly anisotropic case described by the above constitutive equation, there are indeed a great number of additional terms that must be known to characterize a material's electromagnetic response. Fortunately, in practice, a great number of the coefficients in the stiffness/electromagnetic coefficient matrix are zero. MAC/GMC 4.0 admits electromagnetic materials of class C_{6v} with an arbitrary poling direction. These types of materials are in many ways analogous to transversely isotropic materials. Assuming an x_1 poling direction, the above equations reduce to,

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \\ D_1 \\ D_2 \\ D_3 \\ B_1 \\ B_2 \\ B_3 \end{bmatrix}^{(\alpha\beta\gamma)} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 & e_{11} & 0 & 0 & q_{11} & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 & e_{12} & 0 & 0 & q_{12} & 0 & 0 \\ C_{12} & C_{23} & C_{22} & 0 & 0 & 0 & e_{12} & 0 & 0 & q_{12} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 & 0 & 0 & e_{26} & 0 & 0 & q_{26} \\ 0 & 0 & 0 & 0 & 0 & C_{66} & 0 & e_{26} & 0 & 0 & q_{26} & 0 \\ e_{11} & e_{12} & e_{12} & 0 & 0 & 0 & -\kappa_{11} & 0 & 0 & -a_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e_{26} & 0 & -\kappa_{22} & 0 & 0 & -a_{22} & 0 \\ 0 & 0 & 0 & 0 & e_{26} & 0 & 0 & 0 & -\kappa_{22} & 0 & 0 & -a_{22} \\ q_{11} & q_{12} & q_{12} & 0 & 0 & 0 & -a_{11} & 0 & 0 & -\mu_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & q_{26} & 0 & -a_{22} & 0 & 0 & -\mu_{22} & 0 \\ 0 & 0 & 0 & 0 & q_{26} & 0 & 0 & 0 & -a_{22} & 0 & 0 & -\mu_{22} \end{bmatrix}^{(\alpha\beta\gamma)} \begin{bmatrix} \varepsilon_{11} - \varepsilon_{11}^I - \varepsilon_{11}^T \\ \varepsilon_{22} - \varepsilon_{22}^I - \varepsilon_{22}^T \\ \varepsilon_{33} - \varepsilon_{33}^I - \varepsilon_{33}^T \\ 2\varepsilon_{23} - 2\varepsilon_{23}^I - 2\varepsilon_{23}^T \\ 2\varepsilon_{13} - 2\varepsilon_{13}^I - 2\varepsilon_{13}^T \\ 2\varepsilon_{12} - 2\varepsilon_{12}^I - 2\varepsilon_{12}^T \\ -E_1 - E_1^T \\ -E_2 - E_2^T \\ -E_3 - E_3^T \\ -H_1 - H_1^T \\ -H_2 - H_2^T \\ -H_3 - H_3^T \end{bmatrix}^{(\alpha\beta\gamma)}$$

$$\begin{bmatrix} \varepsilon_{11}^T & \varepsilon_{22}^T & \varepsilon_{33}^T & 2\varepsilon_{23}^T & 2\varepsilon_{13}^T & 2\varepsilon_{12}^T & E_1^T & E_2^T & E_3^T & H_1^T & H_2^T & H_3^T \end{bmatrix}^{(\alpha\beta\gamma)} =$$
$$\begin{bmatrix} \alpha_{11} & \alpha_{22} & \alpha_{22} & 0 & 0 & 0 & \zeta_1 & \zeta_2 & \zeta_2 & \psi_1 & \psi_2 & \psi_2 \end{bmatrix}^{(\alpha\beta\gamma)} \Delta T$$

Thus, in addition to the standard thermo-mechanical material coefficients, for an electromagnetic material, the additional material parameters are: three piezoelectric parameters (e_{11}, e_{12}, e_{26}), three piezomagnetic parameters (q_{11}, q_{12}, q_{26}), two dielectric parameters (κ_{11}, κ_{22}), two magnetoelectric parameters (a_{11}, a_{12}), two magnetic permeability parameters (μ_{11}, μ_{22}), two pyroelectric parameters (ζ_1, ζ_2), and two pyromagnetic parameters (ψ_1, ψ_2), for a total of sixteen additional material parameters. In MAC/GMC 4.0, these parameters are read directly from the input file for an assumed x_1 poling direction. However, thanks to MAC/GMC 4.0's transversely isotropic elastic constitutive model that allows an arbitrary direction of transverse isotropy, the actual poling direction of the material is arbitrary.

The present example problem considers a particulate composite consisting of a $BaTiO_3$ (barium titanate) inclusion and a $CoFe_2O_4$ (cobalt ferrite) matrix. The $BaTiO_3$ is a piezoelectric material (non-zero e_{ij} parameters), while the $CoFe_2O_4$ is a piezomagnetic material (non-zero q_{ij} terms). When combined to form a composite, the resulting $BaTiO_3/CoFe_2O_4$ material is electromagnetic. Composites with one or more piezoelectric or electromagnetic phases are often referred to as "smart composites". The smart composite is subjected to strain-controlled loading at 24 °C.

<u>MAC/GMC Input File:</u> `example_7b.mac`

```
MAC/GMC 4.0 Example 7b - Electromagnetic RUC analysis
*ELECTROMAG
*CONSTITUENTS
  NMATS=2
```

```
# -- BaTiO3 (Barium Titanate)
  M=1 CMOD=9 MATID=U MATDB=1 EM=1
  NTP=2
  TEM=24.,600.
  EA=111.93E9,111.93E9
  ET=116.33E9,116.33E9
  NUA=0.321,0.321
  NUT=0.307,0.307
  GA=43.0E9,43.0E9
  ALPA=1.99E-6,1.99E-6
  ALPT=8.53E-6,8.53E-6
  D=0.,0.,1.
  ES11=18.6,18.6
  ES12=-4.4,-4.4
  ES26=11.6,11.6
  QS11=0.0,0.0
  QS12=0.0,0.0
  QS26=0.0,0.0
  KS11=12.6E-9,12.6E-9
  KS22=11.2E-9,11.2E-9
  AS11=0.0,0.0
  AS22=0.0,0.0
  MS11=10.0E-6,10.0E-6
  MS22=5.0E-6,5.0E-6
  PELS1=0.13E5,0.13E5
  PELS2=0.13E5,0.13E5
  PMGS1=0.0,0.0
  PMGS2=0.0,0.0
# -- CoFe2O4 (Cobalt Ferrite)
  M=2 CMOD=9 MATID=U MATDB=1 EM=1 &
  EL=143.57E9,154.57E9,0.37,0.368,45.3E9,0.00E-6,0.00E-6 &
  D=0.,0.,1.
  ES=0.0,0.0,0.0
  QS=699.7,580.3,550.
  KS=0.93E-10,0.08E-9
  AS=0.0,0.0
  MS=157.E-6,-590.E-6
  PELS=0.0,0.0
  PMGS=0.0,0.0
*RUC
  MOD=3 ARCHID=1 VF=0.25 ASP=1. F=1 M=2
*MECH
  LOP=99
  NPT=2 TI=0.,200. MAG=0.,0. MODE=2
  NPT=2 TI=0.,200. MAG=0.,0. MODE=2
  NPT=2 TI=0.,200. MAG=0.,0.02 MODE=1
  NPT=2 TI=0.,200. MAG=0.,0. MODE=2
  NPT=2 TI=0.,200. MAG=0.,0. MODE=2
  NPT=2 TI=0.,200. MAG=0.,0. MODE=2
  NPT=2 TI=0.,200. MAG=0.,0. MODE=2
  NPT=2 TI=0.,200. MAG=0.,0. MODE=2
  NPT=2 TI=0.,200. MAG=0.,0. MODE=2
  NPT=2 TI=0.,200. MAG=0.,0. MODE=2
  NPT=2 TI=0.,200. MAG=0.,0. MODE=2
  NPT=2 TI=0.,200. MAG=0.,0. MODE=2
*THERM
  NPT=2 TI=0.,200. TEMP=24.,24.
```

```
*SOLVER
  METHOD=1 NPT=2 TI=0.,200. STP=100.
*PRINT
  NPL=8
*XYPLOT
  FREQ=1
  MACRO=2
   NAME=example_7b_se X=3 Y=9
   NAME=example_7b_De X=3 Y=46
   NAME=example_7b_Be X=3 Y=49
  MICRO=0
*END
```

Annotated Input Data

1) Flags:
  a) Perform electromagnetic analysis (***ELECTROMAG**) [KM_1]:

2) Constituent materials (***CONSTITUENTS**) [KM_2]:

```
*CONSTITUENTS
  NMATS=2
# -- BaTiO3 (Barium Titanate)
  M=1 CMOD=9 MATID=U MATDB=1 EM=1
  NTP=2
  TEM=24.,600.
  EA=111.93E9,111.93E9
  ET=116.33E9,116.33E9
  NUA=0.321,0.321
  NUT=0.307,0.307
  GA=43.0E9,43.0E9
  ALPA=1.99E-6,1.99E-6
  ALPT=8.53E-6,8.53E-6
  D=0.,0.,1.
  ES11=18.6,18.6
  ES12=-4.4,-4.4
  ES26=11.6,11.6
  QS11=0.0,0.0
  QS12=0.0,0.0
  QS26=0.0,0.0
  KS11=12.6E-9,12.6E-9
  KS22=11.2E-9,11.2E-9
  AS11=0.0,0.0
  AS22=0.0,0.0
  MS11=10.0E-6,10.0E-6
  MS22=5.0E-6,5.0E-6
  PELS1=0.13E5,0.13E5
  PELS2=0.13E5,0.13E5
  PMGS1=0.0,0.0
  PMGS2=0.0,0.0
```

Number of materials:	2	(`NMATS=2`)
Constitutive model:	Arbitrary transversely isotropic	(`CMOD=9`)
Materials:	User-Defined	(`MATID=U`)

Material property source:	Read from input file	(MATDB=1)
Electromagnetic specifier:	Material is electromagnetic	(EM=1)

☞ Note: Electromagnetic analysis requires that the material properties be specified by the user in the MAC/GMC 4.0 input file. The electromagnetic specifier indicates whether or not the particular material has electromagnetic properties (EM=0 indicated that the material does not have electromagnetic properties). Further, the arbitrary transversely isotropic elastic constitutive model (CMOD=9) must be employed.

For illustrative purposes, the material properties for the barium titanate material have been input as temperature-dependent (although the same material properties are employed for each input temperature). As indicated, the sixteen additional material parameters are specified for the two input temperatures. ES11, ES12, and ES26 are the three piezoelectric parameters, QS11, QS12, and QS26 are the three piezomagnetic parameters, KS11 and KS22 are the two magnetoelectric parameters, AS11 and AS22 are the two magnetoelectric parameters, and MS11 and MS22 are the two magnetic permeability parameters. PELS1 and PELS2 correspond to the two pyroelectric parameters (ζ_1, ζ_2), while PMGS1 and PMGS2 correspond to the two pyromagnetic parameters (ψ_1, ψ_2). In addition, a direction vector (D=0.,0.,1.) has been specified. This direction vector is standard input associated with the arbitrary transversely isotropic elastic constitutive model (CMOD=9). It specifies the direction of transverse isotropy with respect to the RUC coordinate axes (i.e., it specifies the vector that is normal to the plane of transverse isotropy) (see the MAC/GMC 4.0 Keywords Manual Section 2). When employed in conjunction with electromagnetic analysis, the direction vector also specifies the poling direction for the electromagnetic material. Thus, in the present case, an x_3 poling direction has been specified for the materials.

```
# -- CoFe2O4 (Cobalt Ferrite)
  M=2 CMOD=9 MATID=U MATDB=1 EM=1 &
  EL=143.57E9,154.57E9,0.37,0.368,45.3E9,0.00E-6,0.00E-6 &
  D=0.,0.,1.
  ES=0.0,0.0,0.0
  QS=699.7,580.3,550.
  KS=0.93E-10,0.08E-9
  AS=0.0,0.0
  MS=157.E-6,-590.E-6
  PELS=0.0,0.0
  PMGS=0.0,0.0
```

For the cobalt ferrite material, temperature-independent material properties have been input. The ordering for the electromagnetic material parameter input is intuitive. For more information on the electromagnetic material parameter specification, see the MAC/GMC Keywords Manual Section 2.

3) Analysis type (***RUC**) → Repeating Unit Cell Analysis [KM_3]:

Analysis model:	Triply periodic GMC	(MOD=3)
RUC architecture:	short fiber, square array	(ARCHID=1)
Fiber volume fractions:	0.25	(VF=0.25)
Fiber aspect ratio	1.	(ASP=1.)
Material assignment:	$BaTiO_3$ (barium titanate) fiber	(F=1)
	$CoFe_2O_4$ (cobalt ferrite) matrix	(M=2)

☞ Note: Electromagnetic analysis requires use of a triply periodic RUC (MOD=3).

4) Loading:
a) Mechanical (***MECH**) [KM_4]:

Loading option: general loading (LOP=99)

Component #1 (ε_{11} or σ_{11})
Number of points: 2 (NPT=2)

Times (TI=) (sec.)	0.	200.
Magnitudes (MAG=)	0.	0.
Control (MODE=)	stress	

Component #2 (ε_{22} or σ_{22})
Number of points: 2 (NPT=2)

Times (TI=) (sec.)	0.	200.
Magnitudes (MAG=)	0.	0.
Control (MODE=)	stress	

Component #3 (ε_{33} or σ_{33})
Number of points: 2 (NPT=2)

Times (TI=) (sec.)	0.	200.
Magnitudes (MAG=)	0.	0.02
Control (MODE=)	strain	

Component #4 (γ_{23} or σ_{23})
Number of points: 2 (NPT=2)

Times (TI=) (sec.)	0.	200.
Magnitudes (MAG=)	0.	0.
Control (MODE=)	stress	

Component #5 (γ_{13} or σ_{13})
Number of points: 2 (NPT=2)

Times (TI=) (sec.)	0.	200.
Magnitudes (MAG=)	0.	0.
Control (MODE=)	stress	

Component #6 (γ_{12} or σ_{12})
Number of points: 2 (NPT=2)

Times (TI=) (sec.)	0.	200.
Magnitudes (MAG=)	0.	0.
Control (MODE=)	stress	

Component #7 (E_1 or D_1)
Number of points: 2 (`NPT=2`)

Times (`TI=`) (sec.)	0.	200.
Magnitudes (`MAG=`)	0.	0.
Control (`MODE=`)	electric displacement (D_1)	

Component #8 (E_2 or D_2)
Number of points: 2 (`NPT=2`)

Times (`TI=`) (sec.)	0.	200.
Magnitudes (`MAG=`)	0.	0.
Control (`MODE=`)	electric displacement (D_2)	

Component #9 (E_3 or D_3)
Number of points: 2 (`NPT=2`)

Times (`TI=`) (sec.)	0.	200.
Magnitudes (`MAG=`)	0.	0.
Control (`MODE=`)	electric displacement (D_3)	

Component #10 (H_1 or B_1)
Number of points: 2 (`NPT=2`)

Times (`TI=`) (sec.)	0.	200.
Magnitudes (`MAG=`)	0.	0.
Control (`MODE=`)	magnetic flux density (B_1)	

Component #11 (H_2 or B_2)
Number of points: 2 (`NPT=2`)

Times (`TI=`) (sec.)	0.	200.
Magnitudes (`MAG=`)	0.	0.
Control (`MODE=`)	magnetic flux density (B_2)	

Component #12 (H_3 or B_3)
Number of points: 2 (`NPT=2`)

Times (`TI=`) (sec.)	0.	200.
Magnitudes (`MAG=`)	0.	0.
Control (`MODE=`)	magnetic flux density (B_3)	

For electromagnetic analysis, the general loading option must be selected. Further, there are now six additional load components that may be applied to the composite: three components of electric displacement or electric field and three components of magnetic flux density or magnetic field. The present case simulates application of strain in the x_3-direction to a composite that is otherwise free of loading. As usual, these "free" loading conditions take the form of zero applied global stresses for all components other than the normal 33 component. The non-loaded strain components are permitted to arise naturally. In terms of electromagnetic effects, "free" loading conditions correspond to zero applied electrical displacement and magnetic flux density (rather than zero applied electric and magnetic field components). The electric and magnetic field components are permitted to arise naturally. For more information on loading specification for electromagnetic analysis, see the MAC/GMC Keywords Manual Section 4.

b) Thermal (***THERM**) [KM_4]:

Number of points:	2	`(NPT=2)`
Time points:	0., 200. sec.	`(TI=0.,200.)`
Temperature points:	24., 24.	`(TEMP=24.,24.)`

c) Time integration (***SOLVER**) [KM_4]:

Time integration method:	Forward Euler	`(METHOD=1)`
Number of points:	2	`(NPT=2)`
Time points:	0., 200. sec.	`(TI=0.,200.)`
Time step size:	100. sec.	`(STP=100)`

☞ Note: Since this problem is linear electro-magneto-elastic, the time step size is unimportant in terms of convergence.

5) Damage and Failure: None

6) Output:

a) Output file print level (***PRINT**) [KM_6]:

Print level:	8	`(NPL=8)`

b) x-y plots (***XYPLOT**) [KM_6]:

Frequency:	1	`(FREQ=1)`
Number of macro plots:	3	`(MACRO=3)`
Macro plot name:	example_7b_se	`(NAME=example_7b_se)`
	example_7b_De	`(NAME=example_7b_De)`
	example_7b_Be	`(NAME=example_7b_Be)`
Macro plot x-y quantities:	ε_{33}, σ_{33}	`(X=3 Y=9)`
	ε_{33}, D_3	`(X=3 Y=46)`
	ε_{33}, B_3	`(X=3 Y=49)`
Number of micro plots:	0	`(MICRO=0)`

7) End of file keyword: (***END**)

Results

Since the particulate $BaTiO_3/CoFe_2O_4$ electromagnetic (smart) composite considered in this example problem consists of phases that are both linear electro-magneto-elastic, results in the form of effective properties (from the MAC/GMC 4.0 output file) will be examined. First, consider the effective (global) electro-magneto-elastic matrix for the smart composite:

```
ZG - Effective/Macro Stiffness/Electromagnetic Coefficient Matrix
     |  2.449D+11  1.382D+11  1.353D+11  0.000D+00  0.000D+00  0.000D+00  0.000D+00  0.000D+00 -1.565D-02  0.000D+00  0.000D+00  3.078D+02 |
     |  1.382D+11  2.449D+11  1.353D+11  0.000D+00  0.000D+00  0.000D+00  0.000D+00  0.000D+00 -1.565D-02  0.000D+00  0.000D+00  3.078D+02 |
     |  1.353D+11  1.353D+11  2.404D+11  0.000D+00  0.000D+00  0.000D+00  0.000D+00  0.000D+00  9.518D-02  0.000D+00  0.000D+00  3.861D+02 |
     |  0.000D+00  0.000D+00  0.000D+00  4.733D+10  0.000D+00  0.000D+00  0.000D+00  4.891D-02  0.000D+00  0.000D+00  3.397D+02  0.000D+00 |
     |  0.000D+00  0.000D+00  0.000D+00  0.000D+00  4.733D+10  0.000D+00  4.891D-02  0.000D+00  0.000D+00  3.397D+02  0.000D+00  0.000D+00 |
     |  0.000D+00  0.000D+00  0.000D+00  0.000D+00  0.000D+00  5.306D+10  0.000D+00  0.000D+00  0.000D+00  0.000D+00  0.000D+00  0.000D+00 |
     |  0.000D+00  0.000D+00  0.000D+00  0.000D+00  4.891D-02  0.000D+00 -1.332D-10  0.000D+00  0.000D+00  2.179D-10  0.000D+00  0.000D+00 |
     |  0.000D+00  0.000D+00  0.000D+00  4.891D-02  0.000D+00  0.000D+00  0.000D+00 -1.332D-10  0.000D+00  0.000D+00  2.179D-10  0.000D+00 |
     | -1.565D-02 -1.565D-02  9.518D-02  0.000D+00  0.000D+00  0.000D+00  0.000D+00  0.000D+00 -1.547D-10  0.000D+00  0.000D+00 -1.210D-11 |
     |  0.000D+00  0.000D+00  0.000D+00  0.000D+00  3.397D+02  0.000D+00  2.179D-10  0.000D+00  0.000D+00  3.517D-04  0.000D+00  0.000D+00 |
     |  0.000D+00  0.000D+00  0.000D+00  3.397D+02  0.000D+00  0.000D+00  0.000D+00  2.179D-10  0.000D+00  0.000D+00  3.517D-04  0.000D+00 |
     |  3.078D+02  3.078D+02  3.861D+02  0.000D+00  0.000D+00  0.000D+00  0.000D+00  0.000D+00 -1.210D-11  0.000D+00  0.000D+00 -1.011D-04 |
```

This matrix relates the global stress/electric displacement/magnetic flux density vector to the global strain/electric field/magnetic field vector, and is of identical form to that of each phase as shown in the local constitutive equation given earlier in this example problem. The composite, like the barium titanate inclusion, has non-zero piezoelectric parameters (e_{ij}). In addition, like the cobalt ferrite matrix, the composite has non-zero piezomagnetic parameters (q_{ij}). Most interesting, however, is the fact that the smart composite has non-zero magnetoelectric parameters (a_{ij}), whereas the a_{ij} of each constituent is zero. These magnetoelectric parameters provide coupling between the electric and magnetic effects in the composite. That is, for instance, when this coupling is present, electric displacement components can arise due to an applied magnetic field. It is noteworthy that the smart composite exhibits this electro-magnetic coupling when neither constituent does.

Next, the effective engineering properties of the smart composite, along with the thermal coefficients, are output:

```
 Effective Engineering Moduli

     E11S=  0.1459E+12
     N12S=  0.3679
     E22S=  0.1459E+12
     N23S=  0.3557
     E33S=  0.1449E+12
     G23S=  0.4733E+11
     G13S=  0.4733E+11
     G12S=  0.5306E+11


 Effective Thermal Expansion Coefficients

   NOTE: Shear CTEs are "engineering" shear CTEs

    0.1927E-05  0.1927E-05  0.3256E-06
    0.3074E-08  0.3074E-08  0.0000E+00


 Effective Pyroelectric Vector

    0.5223E+04  0.5223E+04  0.6427E+04


 Effective Pyromagnetic Vector

   -0.1131E-02 -0.1131E-02  0.2688E+01
```

Here again, the electromagnetic coupling gives rise to interesting effects in the smart composite that are absent in both of the constituents. First of all, shear coefficients of thermal expansion arise in the composite. This indicates that in response to an applied temperature change, global shear strain will result in the composite. Further, while the pyromagnetic parameters for each constituent are zero, the composite has non-zero pyromagnetic parameters. Thus, thermo-magnetic field components (H_k^T) will arise in the smart composite due to applied thermal loading even though this would not occur in either of the monolithic constituents.

Finally, the present example problem involved application of a global normal strain to the smart composite. The time-based output written to the MAC/GMC 4.0 output file includes:

```
     2 TIME:  2.0000D+02     TEMP:  2.4000D+01     TSTEP:  1.0000D+02
 -----------------------------------------------------------------------------------------------
           STRESS:  0.0000D+00   0.0000D+00   2.9041D+09   0.0000D+00   0.0000D+00   0.0000D+00
         E-M FLUX:  0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00
           STRAIN: -7.0886D-03  -7.0886D-03   2.0000D-02   0.0000D+00   0.0000D+00   0.0000D+00
        E-M FIELD:  0.0000D+00   0.0000D+00  -1.3739D+07   0.0000D+00   0.0000D+00  -3.3223D+04
       TH. STRAIN:  0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00
  PYRO E-M FIELD:  0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00

      TANGENT CTE:   1.9274D-06   1.9274D-06   3.2555D-07   3.0735D-09   3.0735D-09   0.0000D+00
       SECANT CTE:   1.9274D-06   1.9274D-06   3.2555D-07   3.0735D-09   3.0735D-09   0.0000D+00
TAN PYRO E-M VECT:   5.2231D+03   5.2231D+03   6.4268D+03  -1.1308D-03  -1.1308D-03   2.6880D+00
SEC PYRO E-M VECT:   5.2231D+03   5.2231D+03   6.4268D+03  -1.1308D-03  -1.1308D-03   2.6880D+00
       NOTE: TREF =  24.000
```

Thus, in response to the applied normal strain loading (ε_{33}), the composite experiences the standard mechanical response (non-zero σ_{33}, ε_{11}, and ε_{22}), in addition to a non-zero electric field component (E_3 = -1.3739×10^7 V/m) and a non-zero magnetic field component (H_3 = -3.3223×10^4 A/m). The S.I. units for the electric field (E_k) are volts per meter (V/m) or Newtons per Coulomb (N/C) and for magnetic field (H_k), the S.I. units are Amperes per meter (A/m). The S.I units for electric displacement (D_k) are Coulombs per square meter (C/m^2) and for magnetic flux density (B_k), the S.I. units are Newtons per Ampere-meter (N/Am). For additional information on electromagnetic analysis with MAC/GMC 4.0, see the MAC/GMC 4.0 Keywords Manual Section 2.

Finally, the local electric and magnetic field components that are induced in the $BaTiO_3/CoFe_2O_4$ composite are shown in Figure 7.3. A relatively constant electric field is induced while the induce magnetic field varies widely in the composite.

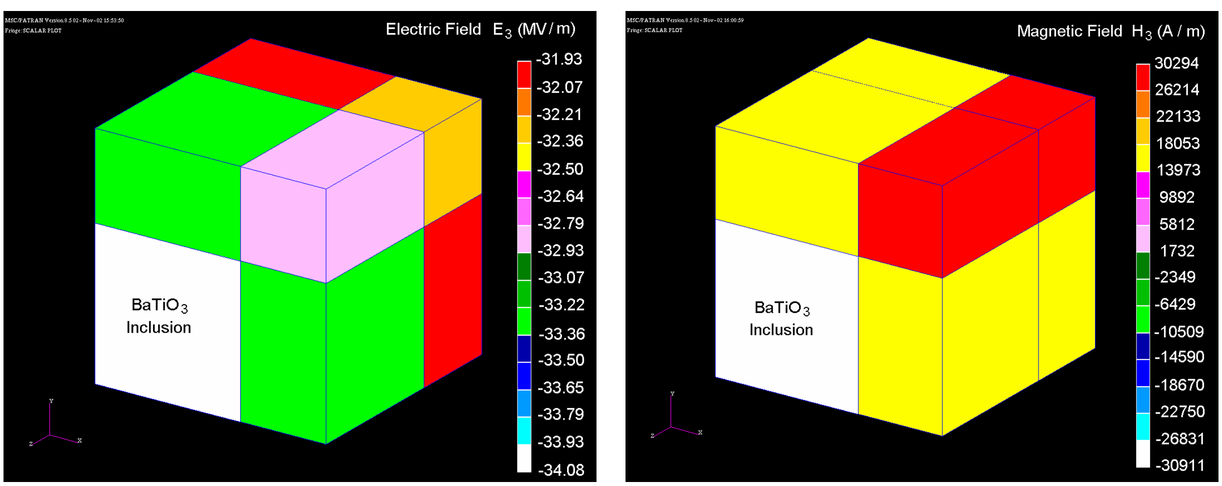

Figure 7.3 Example 7b: Local electric field component E_3 and magnetic field component H_3 induced in the smart $BaTiO_3/CoFe_2O_4$ composite by an applied global strain of $\bar{\varepsilon}_{33} = 0.02$.